

# Breaking of Spatial Diffeomorphism Invariance, Inflation and the Spectrum of Cosmological Perturbations

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Standard inflationary models yield a characteristic signature of a primordial power spectrum with a red tensor and scalar tilt. Nevertheless, Cannone et al [1] recently suggested that, by breaking the assumption of spatial diffeomorphism invariance in the context of the effective field theory of inflation, a blue tensor spectrum can be achieved without violating the Null Energy Condition. In this context, we explore in which cases a blue tensor tilt can be obtained along with a red tilt in the scalar spectrum. Ultimately, we analyze under which conditions this model can reproduce the specific consistency relation of String Gas Cosmology.

## I. INTRODUCTION

One of the key predictions of the inflationary universe scenario [2] is that there should be a nearly scale-invariant spectrum of gravitational waves with a *red tilt*, i.e. slightly more power on larger wavelengths than on smaller ones [3]. This prediction stems from the basic setup of inflation: coupling of a scalar matter field obeying the Null Energy Condition to Einstein gravity. In this setup, the Hubble expansion parameter  $H$  is a slowly decreasing function of time during the period of slow-roll inflation. The amplitude of the gravitational wave spectrum on a scale  $k$  is set by the value of  $H$  at the time when that mode exits the Hubble radius  $H^{-1}$ . Hence, short wavelength modes which exit the Hubble radius at a later time obtain a smaller amplitude of the gravitational wave spectrum [4, 5]. If  $P_T(k)$  denotes the dimensionless power spectrum of gravitational waves as a function of comoving wavenumber  $k$ , and the index  $n_T$  is defined by

$$P(k) \sim k^{n_T}, \quad (1)$$

then the prediction of standard inflation is

$$n_T < 0, \quad (2)$$

i.e. a red tilt. In fact, in the case of single field slow-roll inflation with slow roll parameter  $\epsilon > 0$ , the index is given by (see e.g. the review [6])

$$n_T = -2\epsilon. \quad (3)$$

However, inflation is not the only early universe scenario which is consistent with the current data on cosmic microwave background (CMB) anisotropies and large-scale structure (see e.g. [7] for a recent review of some alternatives). String Gas Cosmology [8–10] is an alternative early universe scenario which follows from merging basic principles of superstring theory with ideas from

cosmology. String Gas Cosmology predicts a spectrum of cosmological perturbations with a blue tilt,

$$n_T > 0. \quad (4)$$

In fact, in the toy model of String Gas Cosmology developed in [8, 9], there is a consistency relation between the index  $n_s$  of the scalar spectrum (which corresponds to a red spectrum) and that of the tensor modes [11, 12], which is given by

$$n_T \simeq -(n_s - 1). \quad (5)$$

Current observations indicate that  $n_s = 0.96 \pm 0.01$  [13]. Assuming that the ratio  $r$  of the tensor spectrum to the scalar spectrum is not too small ( $r > 0.05$ ), then future CMB polarization measurements have the potential to differentiate between the single field inflationary consistency relation (3) and the String Gas consistency relation (5) [14, 15].

In the context of inflation, whereas the index  $n_s$  of the scalar spectrum can be made larger than 1 for certain ranges of wavenumbers by introducing more complicated scalar field actions, the tensor spectrum will remain red, i.e. (2) will remain true. The relation (2) can be violated by abandoning the requirement that matter satisfy the Null Energy Condition. An example is G-inflation [16]. However, the concern is that such models might not be embeddable in an ultraviolet complete theory of matter and gravity [17].

Recently, the suggestion was made [1] that a blue tensor spectrum may result if one abandons some of the symmetries usually taken for granted in inflationary cosmology. In single field inflation the temporal diffeomorphism invariance is broken due to the time-dependence of the background. It might very well be possible, however, that in the Lagrangian of fluctuations also the spatial diffeomorphism is broken. This possibility was considered in [1] in the context of the effective field theory of inflation.

The effective field theory approach corresponds to the description of a system through the lowest dimension operators compatible with the underlying symmetries. This

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theory has been applied, in the last years, to describe the theory of fluctuations around an inflating cosmological background [18]. This approach allows us to characterize all the possible high energy corrections to simple slow-roll inflation, whose sizes are constrained by experiment. Also, it has the advantage of describing in a common language all single field models of inflation by using only symmetry principles.

It was recently shown [1] that the extra terms in the effective field theory Lagrangian which arise if one allows for the breaking of spatial diffeomorphism invariance can produce a blue tilt of the tensor spectrum without assuming the presence of matter which violates the Null Energy Condition. In this note we wish to explore whether it is possible to produce a blue tilt in the tensor spectrum (Section II) while maintaining a red tilt in the scalar spectrum (Section III). More specifically, we wish to determine if it is possible to obtain the consistency relation (5) of String Gas Cosmology in this context.

## II. THE TENSOR SPECTRUM

We are going to follow the approach of [1] and consider the effective field theory for cosmological perturbations around a de Sitter background, with temporal and spatial diffeomorphism invariance broken. Despite the breaking of spatial diffeomorphism invariance, in this scenario isotropy is preserved. The case in which anisotropies can be generated was considered in the recent paper [19].

The procedure usually adopted in the effective theory considers that the scalar mode can be eaten by the metric by going to unitary gauge. In this case there is no perturbation of the inflaton field and all the three degrees of freedom<sup>1</sup> are in the metric (the scalar mode and the two tensor helicities). This setup is analogous to what happens in a spontaneously broken gauge theory. For simplicity we will focus on operators at most quadratic in the fluctuations.

We start with the background metric

$$ds^2 = \bar{g}_{\mu\nu}(\eta)dx^\mu dx^\nu = a^2(\eta)(-\eta_{\mu\nu}dx^\mu dx^\nu), \quad (6)$$

where  $\eta$  is conformal time, the  $x_i$  are comoving spatial coordinates,  $a^2(\eta)$  is the scale factor and  $a(\eta) = 1/(-H\eta)$  for a background de Sitter space. The perturbed metric is

$$ds^2 = g_{\mu\nu}(x, \eta)dx^\mu dx^\nu, \quad (7)$$

and the metric fluctuations are defined by

$$h_{\mu\nu}(x, \eta) = g_{\mu\nu}(x, \eta) - \bar{g}_{\mu\nu}(\eta) \quad (8)$$

and are assumed to be small in amplitude (compared to the background metric).

The breaking of spatial diffeomorphism invariance will be described through effective mass terms in the action for cosmological perturbations. These terms do not necessarily originate from a theory of massive gravity but simply correspond to the most general way to express quadratic non-derivative operators in the fluctuations that break the spatial diffeomorphism symmetry.

Thus, to the usual Einstein-Hilbert action expanded to second order, we add generic operators with no derivatives that are quadratic in the metric fluctuations  $h_{\mu\nu}$ ,

$$\begin{aligned} S = & \int d^4x \sqrt{-g} M_{pl}^2 [R - 2\Lambda - 2cg^{00}] \\ & + \frac{1}{4} M_{pl}^2 \int d^4x \sqrt{-g} [m_0^2 h_{00}^2 + 2m_1^2 h_{0i}^2 - m_2^2 h_{ij}^2 \\ & + m_3^2 h_{ii}^2 - 2m_4^2 h_{00} h_{ii}]. \end{aligned} \quad (9)$$

The terms in the first line are the only ones which contribute to the homogeneous and isotropic background. The terms linear in the metric fluctuations from the first line vanish if the background is a solution of the equations of motion. The terms quadratic in  $h_{\mu\nu}$  from the first line are those which arise in the usual theory of cosmological perturbations. The new term proportional to  $m_0^2$  breaks time reparametrization invariance and the other mass terms break invariance under spatial diffeomorphisms. In the limit  $m_i \rightarrow 0$  with  $i \neq 0$  the invariance is restored.

We can consider the mass terms in the above equation as arising from couplings between the metric and fields with a time-dependent profile during inflation. As an approximation, we assume that their coefficients are effectively constant in space and time during inflation, while they go to zero after inflation ends. However, a small time-dependence proportional to the slow-roll parameter should be expected for these coefficients.

We can write equation (9) in terms of scalar, vector and tensor perturbations, by decomposing the fluctuations as follows,

$$h_{00} = \psi, \quad (10)$$

$$h_{0i} = u_i + \partial_i v, \quad \text{with } \partial_i u_i = 0,$$

$$h_{ij} = \chi_{ij} + \partial_{(i} S_{j)} + \partial_i \partial_j \sigma + \delta_{ij} \tau, \quad \text{with } \partial_i S_i = \partial_i \chi_{ij} = 0.$$

From the tensor part of the action, Cannone et al. [1] obtained the following spectrum

$$\begin{aligned} P_T &= \frac{2H^2}{\pi^2 M_{pl}^2 c_T} \left( \frac{k}{k_*} \right)^{n_T}, \\ n_T &= -2\epsilon + \frac{2}{3} \frac{m_2^2}{H^2} \left( 1 + \frac{4}{3}\epsilon \right), \end{aligned} \quad (11)$$

to first order in the slow-roll parameter. In the above equation  $c_T = 1$  if we consider only the mass terms previously described, neglecting possible higher derivative terms in the Lagrangian. Note that the parameters  $m_0$ ,

<sup>1</sup> Note that here we are not counting the vector modes.

$m_1$ ,  $m_3$  and  $m_4$  do not appear in the action for tensor perturbations.

We can see from the above equation that, if  $m_2^2/H^2$  is positive and sufficiently larger than the slow-roll parameter, then we obtain a positive tensor spectral index [1]. This is an interesting result since it shows that a blue tensor spectrum like in String Gas Cosmology can be obtained in a modified inflationary setup without violating the Null Energy Condition.

### III. THE SCALAR SPECTRUM

Naively, we could fear that the model considered here would also give a blue scalar spectrum, and hence be in contradiction with current observational constraints. We will now show that this is not the case, the reason being that the action contains more free parameters than simply  $m_2$ . We will now derive the constraints on the parameters  $m_i$  from the scalar spectrum.

Expanding the action (9) to second order in the scalar fluctuations and substituting in it the equation of motion of the auxiliary fields  $\psi$ ,  $v$  and  $\sigma$ , we obtain after some algebra

$$S = M_{pl}^2 \int d^4x \frac{a^2}{H^2} \left[ \frac{(m_0^2 + 2\epsilon H^2)(m_2^2 - m_3^2) + m_4^4}{2(m_2^2 - m_3^2)} \tau'^2 + \epsilon H^2 \tau \nabla^2 \tau - \frac{m_2^2 a^2 H^2 (m_2^2 - 3m_3^2 + (3 + \epsilon)m_4^2)}{m_2^2 - m_3^2} \tau^2 \right], \quad (12)$$

which is a function of a single field  $\tau$ . In deriving the above equation, the parameter  $m_1^2$  was chosen to be zero in order to eliminate degrees of freedom, since in this case it can be shown that no vector modes propagate.

The scalar perturbation  $\tau$  is related to the comoving curvature perturbation through the equation

$$\mathcal{R} = \tau - \frac{\mathcal{H}(\tau' - \mathcal{H}\psi)}{\mathcal{H}' - \mathcal{H}^2}. \quad (13)$$

In unitary gauge, the equation of motion of the auxiliary scalar field  $\psi$  leads to  $\tau' = \mathcal{H}\psi$ , and therefore we have  $\mathcal{R} = \tau$ . It is possible to show that in this model the curvature perturbation is not constant on super-Hubble scales. Moreover, the comoving curvature perturbation  $\mathcal{R}$  and the curvature perturbation on uniform density slices  $\zeta$  do not coincide in the large scale limit, unlike what happens if spatial diffeomorphism invariance is unbroken.

We can normalize  $\tau$  by substituting  $\hat{\tau}^2 = N^2 \tau^2$  in equation (12),  $N^2$  being defined as

$$N^2 \equiv \left( \frac{M_{pl}^2}{H^2} \right) \frac{(m_0^2 + 2\epsilon H^2)(m_2^2 - m_3^2) + m_4^4}{2(m_2^2 - m_3^2)}. \quad (14)$$

Then we can write the action in the simpler form

$$S = \int d^4x a^2 [\hat{\tau}'^2 + c_s^2 (\hat{\tau} \nabla^2 \tau) + a^2 M^2 \hat{\tau}^2], \quad (15)$$

where

$$c_s^2 = 2\epsilon H^2 \frac{(m_2^2 - m_3^2)}{(m_0^2 + 2\epsilon H^2)(m_2^2 - m_3^2) + m_4^4}, \quad (16)$$

(note that in the limit when the extra terms in the action go to zero,  $c_s^2$  goes to 1) and

$$M^2 = \frac{-2m_2^2 H^2 (m_2^2 - 3m_3^2 + (3 + \epsilon)m_4^2)}{(m_0^2 + 2\epsilon H^2)(m_2^2 - m_3^2) + m_4^4}, \quad (17)$$

which vanishes in the limit that the coefficients leading to the breaking of the spatial diffeomorphism invariance go to zero.

We can define a new variable  $v \equiv a\hat{\tau}$  and write the action (15) in terms of this variable. Note that in the limit in which the coefficients which lead to the breaking of spatial diffeomorphism invariance go to zero, we find that  $v$  coincides with the standard Sasaki-Mukhanov variable [20], in terms of which the action has canonical form. In our case, it follows from the Euler-Lagrange equation that

$$v'' + c_s^2 k^2 v - [(1 + 2\epsilon)b + (2 + 3\epsilon)] \frac{v}{\eta^2} = 0, \quad (18)$$

where  $b \equiv M^2/H^2$ . We have considered  $a''/a \approx (2 + 3\epsilon)/\eta^2$ , since  $a \approx -(1 + \epsilon)/H\eta$  during inflation. It is the presence of  $b \neq 0$  which leads to the fact that the comoving curvature fluctuation variable is not constant on super-Hubble scales.

Assuming the Bunch-Davis vacuum as initial condition, we have in the limit of small wavelengths the solution

$$v = e^{-ic_s k \eta} / \sqrt{2c_s k}. \quad (19)$$

While in the large wavelength limit we have the solution

$$v = c_1 \eta^{\frac{1}{2}-\alpha} + c_2 \eta^{\frac{1}{2}+\alpha}, \quad (20)$$

where we defined

$$\alpha \equiv \sqrt{2\epsilon b + 3\epsilon + b + 9/4}. \quad (21)$$

The first term is the growing mode (supposing  $b + 9/4 \geq 0$ ). We are going to consider only this growing mode.

If we match the solution for small and for large wavelengths when  $c_s^2 k^2 = (b + 2)/\eta^2$  [21, 22], we obtain for the constant  $c_1$  the following expression

$$c_1 = \frac{e^{-i\sqrt{b+2}}}{\sqrt{2c_s k}} \left( \frac{c_s k}{\sqrt{b+2}} \right)^{\frac{1}{2}-\alpha}. \quad (22)$$

From these solutions, we obtain the following scalar power spectrum

$$P_{\mathcal{R}} = \frac{k^3 |v|^2}{a^2 N^2} = k^{3-2\alpha} \left( \frac{c_s^{-2\alpha}}{2a^2 N^2} \right) \left( \frac{\eta}{\sqrt{b+2}} \right)^{1-2\alpha}. \quad (23)$$

Since in this model the curvature perturbation is not constant after Hubble radius crossing, an explicit time-dependence appears in the above expression. We need to evaluate the result at the end of inflation ( $\eta = \text{cst}$ ,  $a = \text{cst}$ ).

We can see that the scalar spectral index is given by

$$n_s - 1 = 3 - 2\alpha = 3 - 2\sqrt{2\epsilon b + 3\epsilon + b + 9/4}. \quad (24)$$

Thus, the spectrum will be red,  $n_s < 1$ , if

$$\sqrt{2\epsilon b + 3\epsilon + b + 9/4} > 3/2. \quad (25)$$

We can see that, in order to obtain the observed value for the spectral index,  $n_s = 0.96$ , the parameter  $b$  must be close to zero. This can occur if the condition  $m_2^2 \approx 3m_3^2$  is satisfied, in addition to the condition  $m_4 \approx 0$ , (see eq. (17)). Also we can see from equation (23), which is calculated at a fixed time (the end of inflation), that the expected amplitude can also be obtained if  $c_s \rightarrow 1$  and  $N^2 \rightarrow \epsilon$ . One possibility to obtain these limits is if, in addition to  $m_4 \approx 0$ , the parameter  $m_0$  is much smaller than  $\sqrt{\epsilon}H$ . Although this is not the only possibility that provides the observed value of the scalar spectrum, it is an interesting specific case to be considered. We can see that none of these bounds implies in an upper limit for the parameter  $m_2$ . This proves that its possible in this model to have a blue tensor spectrum and at the same time maintain the observed red scalar spectrum.

Comparing equations (23) and (24) with the corresponding equations in the usual models of inflation, we can see that the slow-roll parameter that appears in the term  $3\epsilon$  in the above equations must correspond to the slow-roll parameter calculated at the moment of Hubble radius exit. When  $b \rightarrow 0$ , the curvature perturbation is conserved after Hubble radius crossing and thus, in this limit, the slow-roll parameter is calculated at the Hubble radius exit. Therefore, we are going to denote this quantity by  $\epsilon_c$ .

We can write a simplified expression for  $n_s - 1$  by expanding the square root in equation (24) as follows,

$$n_s - 1 = 3 - 3\sqrt{\frac{8}{9}\epsilon b + \frac{4}{3}\epsilon_c + \frac{4}{9}b + 1} \approx -2\epsilon_c - \frac{2}{3}b. \quad (26)$$

This quantity must be approximately  $-0.04$  according to observations. Since the usual inflationary models give good agreement with observations for this index, the above expression must correspond to  $n_s - 1 \approx -2\epsilon_V$ , where  $\epsilon_V$  is the slow-roll parameter in the usual inflationary models. Thus, we have the following relation,

$$\epsilon_c + \frac{b}{3} \approx \epsilon_V. \quad (27)$$

From this relation we can see that in the model considered here the value of the slow-roll parameter at the time of Hubble radius crossing can be smaller (or bigger) than the usual one by an amount of  $\approx b/3$ .

Using equations (11) and (23), we can calculate the tensor-to-scalar ratio defined as

$$r \equiv P_t(k_*)/P_S(k_*) = A_t/A_S. \quad (28)$$

We then obtain

$$r = \frac{2H^2}{\pi^2 M_{pl}^2} \left( \frac{2a^2 N^2}{c_s^{-2\alpha}} \right) \left( \frac{\sqrt{b+2}}{\eta} \right)^{1-2\alpha}, \quad (29)$$

where  $N$  and  $c_s$  are given by eqs. (14) and (16) respectively. It is possible to verify that in the limit  $b = 0$ ,  $c_s = 1$  and  $N^2 = \epsilon$ , the expected expression for the tensor-to-scalar ratio is recovered. In this case, the usual consistency relation,  $r = -8n_t$ , is also recovered.

By comparing the expressions (11) and (26) for the tensor and scalar spectral index of the model considered,

$$n_T = -2\epsilon + \frac{2}{3} \frac{m_2^2}{H^2} (1 + 2\epsilon), \quad (30)$$

$$n_s - 1 = -2\epsilon - \frac{2}{3} \frac{M^2}{H^2} (1 + 2\epsilon), \quad (31)$$

it is possible to see that in the case the tensor spectrum is blue, when the second term in the expression (30) is bigger than the first, one can obtain the String Gas Cosmology relation,  $n_t \approx -(n_s - 1)$ . This relation is satisfied whenever

$$-2\epsilon + \frac{2}{3} \frac{m_2^2}{H^2} (1 + 2\epsilon) = +2\epsilon + \frac{2}{3} \frac{M^2}{H^2} (1 + 2\epsilon). \quad (32)$$

We should point out that  $M^2$  can be positive or negative, but the right hand side of equation (31) must be negative in order to provide a red scalar spectrum.

The above equality can be satisfied for a scalar spectrum compatible with observations with a complementary blue tensor spectrum. Therefore, we conclude that the model considered here can reproduce the characteristic consistency relation of the String Gas Cosmology. Obviously, this requires fine tuning of the parameters of the model.

The results introduced here have been obtained from mass operators in the Lagrangean that break spatial diffeomorphism invariance. Nevertheless, in [1] it was shown that certain operators containing more than two spatial derivatives can mimic the effects of these mass operators even in scenarios which preserve spatial diffeomorphism invariance.

#### IV. DISCUSSION

A primordial tensor power spectrum with a blue spectral tilt is a characteristic signature of some alternative models to inflation, in particular String Gas Cosmology, but also of some inflationary models which violate the Null Energy Condition. A possible detection of a blue tensor tilt would immediately falsify standard inflation



(based on the usual symmetries and on assuming matter which satisfies the Null Energy Condition). Since from the point of view of an ultraviolet complete theory of matter it is problematic to violate the Null Energy Condition [17], it becomes important to carefully investigate different ways of producing a blue tensor tilt.

In this spirit, it has recently been pointed out [1] that the breaking of spatial diffeomorphism invariance during inflation could provide a new mechanism to generate a blue tensor tilt, without postulating matter which violates the Null Energy Condition. On the other hand, its well known that the tilt of the scalar spectrum is observed to be red. Hence, in this paper we have studied under which conditions one can obtain both a blue tensor tilt and a red scalar tilt compatible with current observations. We have seen that this is possible with appropriate choices of the new parameters in the effective Lagrangean.

Assuming that the tensor-to-scalar ratio is not too small ( $r > 0.05$ ), then future CMB polarization measurements have the potential to differentiate the single field inflationary consistency relation,  $n_T = 2\epsilon$ , and the String Gas consistency relation,  $n_T \simeq -(n_s - 1)$  [14, 15]. Motivated by this prospect, we have studied whether it is possible for the models introduced in [1] to produce

scalar and tensor tilts which agree with the consistency relation from String Gas Cosmology. We have seen that, given special choices of the parameters, this is also possible.

Thus, we suggest that its important to investigate other predictions of this scenario, like non-Gaussianities, in order to find which observables could be able to distinguish the special class of models of [1] which are consistent with the string gas consistency relation from the actual String Gas Cosmology scenario. String Gas Cosmology predicts negligible non-Gaussianities on cosmological scales [23], whereas usual inflationary models predict typically small but non-negligible non-Gaussianities. A similar proposal to distinguish various scenarios consistent with a blue tensor tilt has been made in [24].

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